Algebra, Algebra of Expressions

1. $(a+b)^2 = a^2 + 2ab + b^2;\; a^2 + b^2 = (a+b)^2 - 2ab$
2. $(a-b)^2 = a^2 - 2ab + b^2;\; a^2 + b^2 = (a-b)^2 + 2ab$
3. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
4. $(a+b)^3 = a^3 + b^3 + 3ab(a+b);\; a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
5. $(a-b)^3 = a^3 - b^3 - 3ab(a-b);\; a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
6. $a^2 - b^2 = (a+b)(a-b)$
7. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
8. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
9. $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \cdots + b^{n-1})$
10. $a^n = a.a.a \ldots n$ times
11. $a^m.a^n = a^{m+n}$
12. $\dfrac{a^m}{a^n} = a^{m-n}$ if $m > n$

    $= 1$ if $m = n$

    $= \dfrac{1}{a^{n-m}}$ if $m < n; a \in R, a \neq 0$
13. $(a^m)^n = a^{mn} = (a^n)^m$
14. $(ab)^n = a^n.b^n$
15. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
16. $a^0 = 1$ where $a \in R, a \neq 0$
17. $a^{-n} = \dfrac{1}{a^n}, a^n = \dfrac{1}{a^{-n}}$
18. $a^{p/q} = \sqrt[q]{a^p}$
19. If $a^m = a^n$ and $a \neq \pm 1, a \neq 0$ then $m = n$
20. If $a^n = b^n$ where $n \neq 0$, then $a = \pm b$
21. If $\sqrt{x}, \sqrt{y}$ are quadratic surds and if $a + \sqrt{x} = \sqrt{y}$, then $a = 0$ and $x = y$
22. If $\sqrt{x}, \sqrt{y}$ are quadratic surds and if $a + \sqrt{x} = b + \sqrt{y}$ then $a = b$ and $x = y$
23. If a, m, n are positive real numbers and $a \neq 1$, then $\log_a mn = \log_a m + \log_a n$
24. If a, m, n are positive real numbers, $a \neq 1$, then $\log_a \left(\dfrac{m}{n}\right) = \log_a m - \log_a n$
25. If a and m are positive real numbers, $a \neq 1$ then $\log_a m^n = n \log_a m$
26. If a, b and k are positive real numbers, $b \neq 1, k \neq 1$, then $\log_b a = \dfrac{\log_k a}{\log_k b}$
27. $\log_b a = \dfrac{1}{\log_a b}$ where a, b are positive real numbers, $a \neq 1, b \neq 1$
28. if a, m, n are positive real numbers, $a \neq 1$ and if $\log_a m = \log_a n$, then $m = n$

29. if $a+ib=0$ where $i=\sqrt{-1}$, then $a=b=0$
30. if $a+ib=x+iy$, where $i=\sqrt{-1}$, then $a=x$ and $b=y$
31. The roots of the quadratic equation $ax^2+bx+c=0;\ a\neq 0$ are $\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

    The solution set of the equation is $\left\{\dfrac{-b+\sqrt{\Delta}}{2a},\dfrac{-b-\sqrt{\Delta}}{2a}\right\}$

    where $\Delta=$ discriminant $=b^2-4ac$
32. The roots are real and distinct if $\Delta>0$.
33. The roots are real and coincident if $\Delta=0$.
34. The roots are non-real if $\Delta<0$.
35. If α and β are the roots of the equation $ax^2+bx+c=0, a\neq 0$ then
    i) $\alpha+\beta=\dfrac{-b}{a}=-\dfrac{\text{coeff. of } x}{\text{coeff. of } x^2}$
    ii) $\alpha\cdot\beta=\dfrac{c}{a}=\dfrac{\text{constant term}}{\text{coeff. of } x^2}$
36. The quadratic equation whose roots are α and β is $(x-\alpha)(x-\beta)=0$

    i.e. $x^2-(\alpha+\beta)x+\alpha\beta=0$

    i.e. $x^2-Sx+P=0$ where $S=$Sum of the roots and $P=$Product of the roots.
37. For an arithmetic progression (A.P.) whose first term is (a) and the common difference is (d).
    i) n^{th} term$=t_n=a+(n-1)d$
    ii) The sum of the first (n) terms $=S_n=\dfrac{n}{2}(a+l)=\dfrac{n}{2}\{2a+(n-1)d\}$

        where $l=$last term$=a+(n-1)d$.
38. For a geometric progression (G.P.) whose first term is (a) and common ratio is (γ),
    i) n^{th} term$=t_n=a\gamma^{n-1}$.
    ii) The sum of the first (n) terms:

$$\begin{aligned} S_n &= \frac{a(1-\gamma^n)}{1-\gamma} && \text{if}\gamma<1 \\ &= \frac{a(\gamma^n-1)}{\gamma-1} && \text{if } \gamma>1 \\ &= na && \text{if } \gamma=1 \end{aligned}.$$

39. For any sequence $\{t_n\}, S_n-S_{n-1}=t_n$ where $S_n=$Sum of the first (n) terms.
40. $\sum\limits_{\gamma=1}^{n}\gamma=1+2+3+\cdots+n=\dfrac{n}{2}(n+1)$.
41. $\sum\limits_{\gamma=1}^{n}\gamma^2=1^2+2^2+3^2+\cdots+n^2=\dfrac{n}{6}(n+1)(2n+1)$.

42. $\sum_{\gamma=1}^{n} \gamma^3 = 1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \frac{n^2}{4}(n+1)^2.$

43. $n! = (1).(2).(3).\ldots..(n-1).n.$

44. $n! = n(n-1)! = n(n-1)(n-2)! = \ldots..$

45. $0! = 1.$

46. $(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \cdots + b^n, n > 1.$

MENSURATION

Mensuration is a branch of mathematics which deals with the measurements of lengths of lines, areas of surfaces and volumes of solids.

Mensuration may be divided into two parts.

1. Plane mensuration
2. Solid mensuration

Plane mensuration deals with perimeter, length of sides and areas of two dimensional figures and shapes.

Solid mensuration deals with areas and volumes of solid objects.

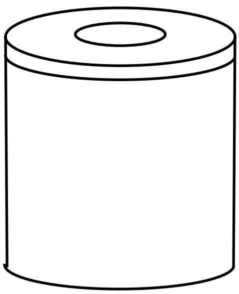
After learning this chapter you will be able to

* Recognise a cylinder, a cone and a sphere.
* Understand the properties of cylinder, cone and sphere.
* Distinguish between the structure of cylinder and cone.
* Derive the formula to find the surface area and volume of cylinder, cone and sphere.
* Solve simple problems pertaining to the surface area and volume of cylinder, cone and sphere.

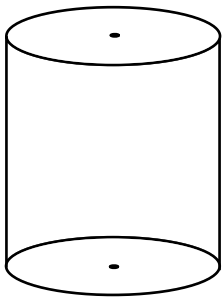
CYLINDER

Observe the following figures :

Road roller



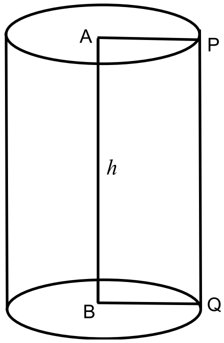
Circular based storage Tank



Cylinder

Wheels of a road roller, a circular based storage tank etc will suggest you, the concept of a right circular cylinder.

1. The right circular cylinder



A right circular cylinder is a solid described by revolution of a rectangle about one of its sides which remains fixed.

AP = Radius of the circular plane

AB = Axis of the cylinder

PQ = Height of the cylinder

Features of a right circular cylinder

1) A right circular cylinder has two plane surfaces, circular in shape.
2) The curved surface joining the plane surfaces is the lateral surface of the cylinder.
3) The two circular planes are parallel to each other and also congruent.
4) The line joining the centers of the circular planes is the axis of the cylinder.
5) All the points on the lateral surface of the right circular cylinder are equidistant from the axis.
6) Radius of circular plane is the radius of the cylinder.

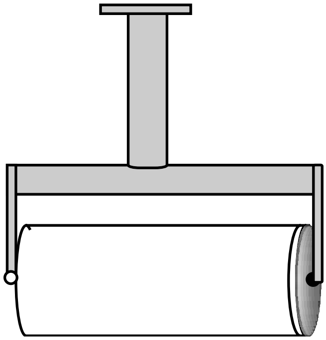
Two types of cylinders :

1. Hollow cylinder and
2. Solid cylinder

A hollow cylinder is formed by the lateral surface only.

Example : A pipe

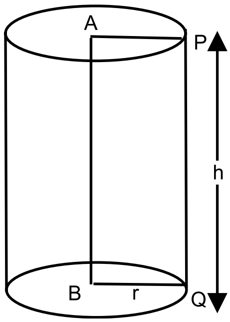
A solid cylinder is the region bounded by two circular plane surfaces and also the lateral surface.



Example : A garden roller

2. Surface area of a right circular cylinder

A. Lateral Surface area :



Activity :

1. Take a strip of paper having width equal to the height of the cylinder.
2. Wrap the strip around the lateral surface of the cylinder and cut the overlapping strip along the vertical line. (say PQ)
3. You will get a rectangular paper cutting which exactly covers the lateral surface.
4. Area of the rectangle is equal to the area of the curved surface of the cylinder.

Expression for the lateral surface area :

(i) Length of the rectangle is equal to the circumference, $l = 2\pi r$

(ii) Breadth of the rectangle b is the height of the cylinder = h

Area of the rectangle $A = l \times b$

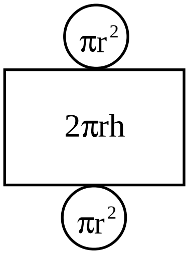
Lateral surface area of the Cylinder $A = 2\pi r h$

$$\boxed{A = 2\pi r h \text{ sq. units}}$$

Observe :

Surface area of a thin hollow cylinder having circumference P and height is 'h' = Ph or = $2\pi r h$ ($\because P = 2\pi r$)

B. Total surface area of a cylinder :



$$\text{The total surface area of the cylinder} = \left[\begin{array}{l}\text{Area of two}\\ \text{circular bases}\end{array}\right] + \left[\begin{array}{l}\text{Lateral surface}\\ \text{area of cylinder}\end{array}\right]$$

$$= \pi r^2 + \pi r^2 + 2\pi rh$$

$$= 2\pi r^2 + 2\pi rh$$

$$= \boxed{2\pi r(r + h)} \text{ sq. units}$$

Lateral surface area of a cylinder = $2\pi rh$ sq. units.

Total surface area of a cylinder = $2\pi r(r+h)$ sq. units.

Remember :

Area is always expressed in square units

Worked examples :

Example 1 **:** Find the lateral surface area of a cylinder whose circumference is 44 cm and height 10 cm.

Given : circumference = $2\pi r$ = 44 cms

and height = h = 10 cms

Solution : Lateral surface area of cylinder $= 2\pi rh$

$= 44 \times 10$

$= \boxed{440 \text{ sq. cm.}}$

Example 2 : Find the total surface area of the cylinder, given that the diameter is 10 cm and height is 12.5 cm.

Given : Diameter = d = 10 cms

$$\therefore\ r = \frac{d}{2} = 5 \text{ cms}$$

and height = h = 12.5 cms.

Solution : Total surface area of a cylinder = $2\pi r\ (r+h)$

$$= 2 \text{x} \frac{22}{7} \text{x} 5 \text{x} (5 + 12.5)$$

$$= 2 \text{x} \frac{22}{7} \text{x} 5 \text{x}\ 17.5$$

$$= \boxed{550 \text{ sq.cm.}}$$

Example 3 : The lateral surface area of a thin circular bottomed tin is 1760 sq. cm and radius is 10 cm. What is the height of the tin?

Given : The lateral surface area of a cylinder = $2\pi rh$ = 1760 sq. cm.

radius = r = 10 cms

Solution : Lateral surface area of a cylinder = $2\pi rh$

$$1760 = 2 \text{x} \frac{22}{7} \text{ x } 10 \text{ x } h$$

$$h = \frac{1760 \text{ x } 7}{2 \text{ x } 22 \text{ x } 10}$$

$$h = \boxed{28 \text{ cms}}$$

Exercise : 9.1

1) The radius of the circular base of a cylinder is 14 cm and height is 10 cm. Calculate the curved surface area of the cylinder.

2) The circumference of a thin hollow cylindrical pipe is 44 cm and length is 20 mts. Find the surface area of the pipe.

3) A cylinder has a diameter 20 cm and height 18 cm. Calculate the total surface area of the cylinder.

4) Lateral surface area of a cylinder is 1056 sq. cm and radius is 14 cm. Find the height of the cylinder.

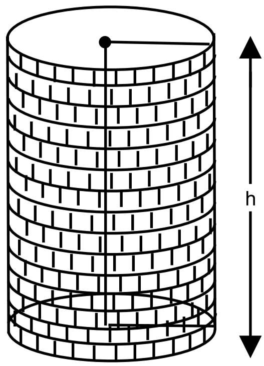
5) A mansion has twelve cylindrical pillars each having the circumference 50 cm and height 3.5 mts. Find the cost of painting the lateral surface of the pillars at Rs. 25 per squaremeter.

6) The diameter of a thin cylindrical vessel open at one end is 3.5 cm and height is 5 cm. Calculate the surface area of the vessel.

7) A closed cylindrical tank is made up of a sheet metal. The height of the tank is 1.3 meters and radius is 70 cm. How many square meters of sheet metal is required to make?

8) A roller having radius 35 cms and length 1 meter takes 200 complete revolutions to move once over a play ground. What is the area of the playground?

3. Volume of a right circular cylinder :



Activity :

1) A coin is placed on a horizontal plane.

2) Pile up the coins of same size one upon the other such that they form a right circular cylinder of height 'h'.

Volume of a cylinder = Bh

The area of the circular base B = πr^2 [B is the circular base of radius r]

Height of the cylinder = h

$\therefore$ Volume of the cylinder = $\pi r^2 h$ cubic units

Volume of the right circular cylinder of radius 'r' and height 'h' = $\pi r^2 h$ cubic units. Volume is expressed in cubic units.

Worked examples :

Example 1 : Find the volume of the cylinder whose radius is 7 cm and height is 12 cm.

Given : Radius of the cylinder = r = 7 cm

Height of the cylinder = h = 12 cm

Solution : Volume of the cylinder V $= Bh$

$= \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 12$

$= \frac{22}{7} \times 49 \times 12$

$= 1848$ cubic cms.

$\therefore$ $\boxed{\text{Volume of the cylinder} = 1848 \text{ cc}}$

Example 2 : Volume of the cylinder is 462 cc and its diameter is 7 cm. Find the height of the cylinder.

Given : Volume = V = 462 cc

Diameter = d = 7 cm $\quad \therefore$ r = 3.5 cm.

Solution : Volume of a cylinder V $= \pi r^2 h$

$$462 = \frac{22}{7} \times (3.5)^2 \times h$$

$$h = \frac{462 \times 7}{22 \times (3.5)^2}$$

$$h = 12 \text{ cm}$$

$\therefore$ $\boxed{\text{Height of the cylinder } h = 12 \text{ cms}}$

Exercise : 9.2

1) Area of the base of a right circular cylinder is 154 sq. cm and height is 10 cm. Calculate the volume of the cylinder.

2) Find the volume of the cylinder whose radius is 5 cm and height is 28 cm.

3) The circumference of the base of a cylinder is 88 cm and its height is 10 cm. Calculate the volume of the cylinder.

4) Volume of a cylinder is 3080 cc and height is 20 cm. Calculate the radius of the cylinder.

5) A cylindrical vessel of height 35 cm contains 11 litres of juice. Find the diameter of the vessel (one litre = 1000 cc.)

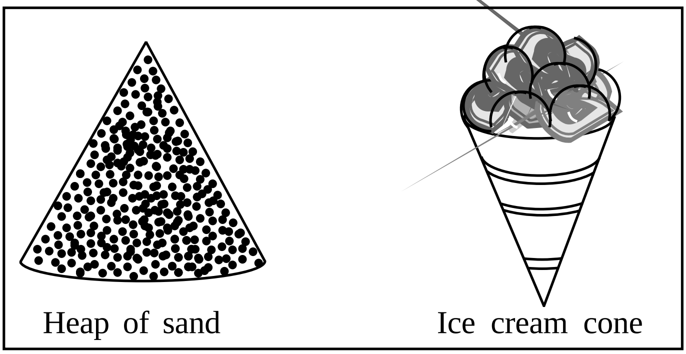
6) Volume of a cylinder is 4400 cc and diameter is 20 cm. Find the height of the cylinder.

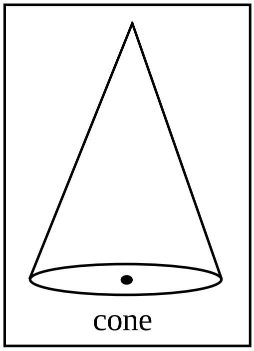
7) The height of water level in a circular well is 7 mts and its diameter is 10 mts. Calculate the volume of water stored in the well.

8) A thin cylindrical tin can hold only one litre of paint. What is the height of the tin if the diameter of the tin is 14 cm? (one litre = 1000 cc)

THE RIGHT CIRCULAR CONE

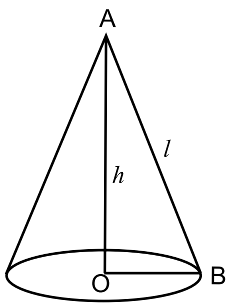
Observe the following figures :





A heap of sand, an Ice cream cone suggests you the concept of a right circular cone.

1. Surface area of a right circular cone :



A right circular cone is a solid generated by the revolution of a right angle triangle about one of the sides containing the right angle.

Properties of a right circular cone :

1) A cone has a circular plane as its base.

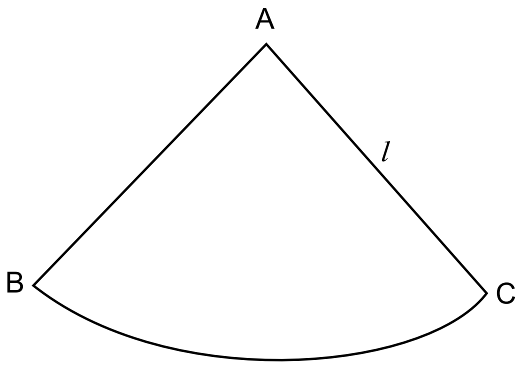
2) The point of intersection of the axis of the cone and slant height is the vertex of the cone (A).

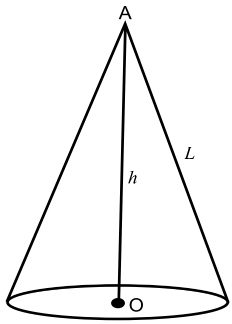
3) The curved surface which connects the vertex and circular edge of the base is the lateral surface of the cone.

4) The line joining the vertex and the center of the circular base is the height of the cone (AO = h)

> **Remember :**
>
> The distance between the vertex and any point on the circumference of the base is the slant height.

A. The curved surface area :





Activity :

1) Take a right circular cone.

2) Wrap the curved surface with a piece of paper.

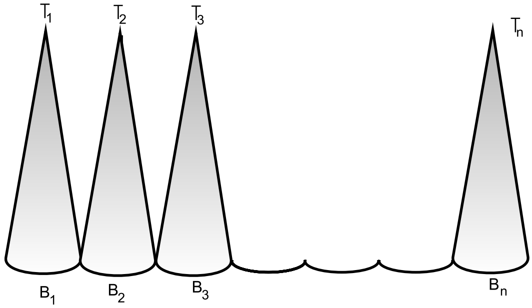
3) Cut the paper along the length of slant height say AB

4) Take out the paper which exactly covers the curved surface.

5) Spread the paper on a plane surface.

> **Observe :**
>
> Radius of the circular section is equal to slant height of the cone = l

This circular section can be divided into small triangles as shown in the figure say T_1, T_2, T_3 ... T_n



From the figure lateral surface area of the circular section = sum of the areas of each triangle.

$$= T_1 + T_2 + \ldots\ldots T_n$$

Area of the Triangle = $\frac{1}{2}$ x base x height

i.e. Area of $T_1 = \frac{1}{2} \times B_1 \times l$

$T_2 = \frac{1}{2} \times B_2 \times l$

$T_3 = \frac{1}{2} \times B_3 \times l$

Area of $T_n = \frac{1}{2} \times B_n \times l$

Area of circular section $= \frac{1}{2} B_1 l + \frac{1}{2} B_2 l + \ldots\ldots\ldots\ldots\ldots + \frac{1}{2} B_n l$

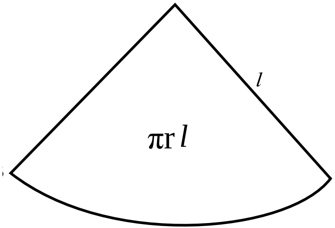
$$= \frac{1}{2} l \left(B_1 + B_2 + \ldots\ldots + B_n\right)$$

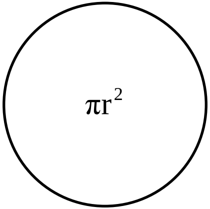
$$= \frac{1}{2} l \left(2\pi r\right) \qquad [B_1 + B_2 + \ldots. + B_n = 2\pi r]$$

$$= \frac{1}{2} l \times 2\pi r$$

∴ Area of the curved surface of a cone = $\pi r l$ sq. units

B. Total Surface area of a cone :





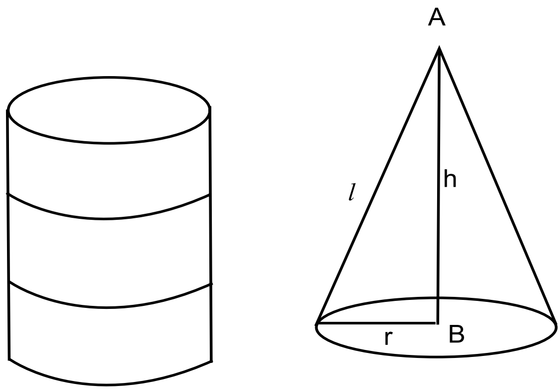
Total surface area of a cone = Area of circular base + Area of the curved surface

$$= \pi r^2 + \pi r l$$

$$= \pi r (r + l)$$

$$\boxed{\text{Total surface area of a cone} = \pi r [r + l] \text{ sq. units}}$$

2. Volume of a right circular cone



Suggested Activity :

1) Take a conical cup and a cylindrical vessel of the same radius and height.
2) Fill the conical cup with water up to its brim.
3) Pour the water into cylindrical vessel.
4) Count how many cups of water is required to fill the cylindrical vessel upto its brim.

Observe that exactly three cups of water is required to fill the vessel.

Volume of a cylinder = 3 x volume of a cone having same base and height.

$\therefore$ Volume of a cone $= \frac{1}{3}$ of the volume of a cylinder having same base and height.

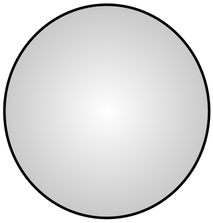
$= \frac{1}{3} \times Bh$ [$\because$ Volume of a cylinder = Bh]

$= \frac{1}{3}\pi r^2 h$ [$\because$ $B = \pi r^2$]

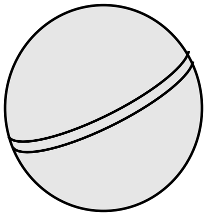
Volume of a cone of radius r and height h = $\frac{1}{3}\pi r^2 h$ cubic units.

THE SPHERE

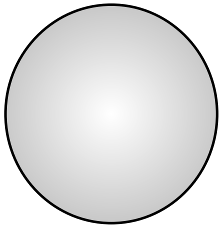
Observe the following figures



Shot put



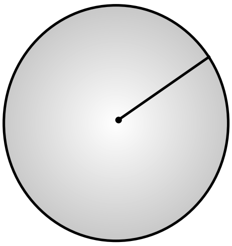
Ball



Sphere

A shotput, a ball etc, will suggest you the concept of a sphere.

Properties of a sphere :



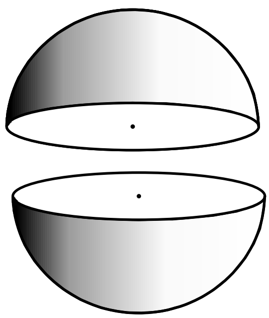
A sphere is a solid described by the revolution of a semi circle about a fixed diameter.

1) A sphere has a centre
2) All the points on the surface of the sphere are equidistant from the centre.
3) The distance between the centre and any point on the surface of the sphere is the radius of the sphere.

Remember :

A plane through the centre of the sphere divides it into two equal parts each called a hemisphere.

1. Surface Area of a sphere :



Activity :

1) Consider a sphere of radius r.
2) Cut the solid sphere into two equal halves.
3) Fix a pin at the top most point of a hemisphere.
4) Starting from the centre point of the curved

surface of the hemisphere, wind a uniform thread so as to cover the whole curved surface of the hemisphere.

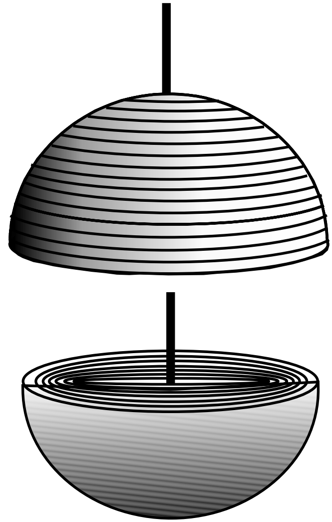
5) Measure the length of the thread.

6) Similarily, fix a pin at the center of the plane circular surface.

7) Starting from the centre, wind the thread of same thickness to cover the whole circular surface.

8) Unwind and measure the lengths of the threads.

9) Compare the lengths.



What do you observe from both the activities?

It is found that the length of the thread required to cover the curved surface is twice the length required to cover the circular plane surface.

Area of the plane circular surface $= \pi r^2$

$\therefore$ Curved surface area of a hemisphere $= 2\pi r^2$

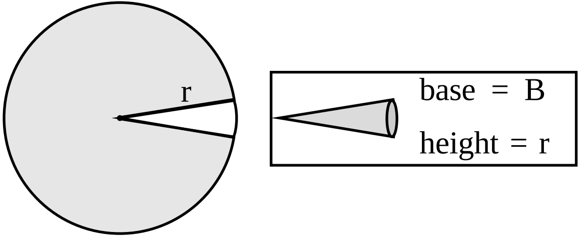
Surface area of the whole sphere $= 2\pi r^2 + 2\pi r^2$

$= 4\pi r^2$

Surface area of a sphere of radius r = $4\pi r^2$ sq. units.

2. Volume of a sphere

Observe the following figures.



A solid sphere is made up of miniature cones whose height is equal to the radius of the sphere and each having circular base.

Volume of each cone $= \frac{1}{3}$ x base x height

Volume of cone 1 $= \frac{1}{3} \times B_1 \times r$

Volume of cone 2 $= \frac{1}{3} \times B_2 \times r$ etc

Volume of cone n $= \frac{1}{3} \times B_n \times r$

Volume of the sphere = Sum of the volumes of all the cones

$$= \frac{1}{3} \times B_1 \times r + \frac{1}{3} \times B_2 \times r + \dots . \frac{1}{3} \times B_n \times r$$

$$= \frac{1}{3} r (B_1 + B_2 + \dots\dots\dots + B_n) \quad [\because \text{T.S.A. of sphere } 4\pi r^2]$$

$$= \frac{1}{3} r (B) \quad [\because \text{Surface area of sphere}]$$

$$= \frac{1}{3} r \times 4\pi r^2$$

$$= \frac{4}{3}\pi r^3$$

$$\boxed{\text{Volume of the sphere} = \frac{4}{3}\pi r^3 \text{ cubic units}}$$

Volume of the hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$$= \frac{2}{3}\pi r^3$$

$$\boxed{\therefore \text{ Volume of a hemisphere } = \frac{2}{3}\pi r^3 \text{ cubic untis}}$$

Remember at a glance :

Solid	Curved Surface area	Total Surface area	Volume
Cylinder	$2\pi rl$	$2\pi r\ (r+l)$	$\pi r^2 h$
Cone	πrl	$\pi r\ (r+l)$	$\frac{1}{3}\pi r^2 h$
Sphere	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Solid hemisphere	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$

Area of triangle	$= \frac{1}{2} \times$ base $\times$ height	$= \frac{1}{2}\, bh$
Area of rectangle	= length x breadth	$= lb$
Area of trapezium	$= \frac{1}{2} \times$ height $\times$ (sum of two parallel sides)	$= \frac{1}{2}\, h\,(a+b)$

3-D Figures:

Prism: $V = Bh$

Pyramid: $V = \frac{1}{3}Bh$

Cylinder: $V = \pi r^2 h$; $SA = 2\pi rh + 2\pi r^2$

Cone: $V = \frac{1}{3}\pi r^2 h$; $SA = s\pi r + \pi r^2$

Sphere: $V = \frac{4}{3}\pi r^3$; $SA = 4\pi r^2 = \pi d^2$

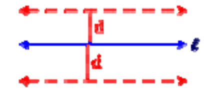
Regular Solids:

Tetrahedron – 4 faces
Cube – 6 faces
Octahedron – 8 faces
Dodecahedron – 12 faces
Icosahedron – 20 faces

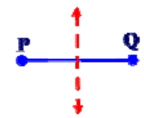
Locus Theorems:

Fixed distance from point.

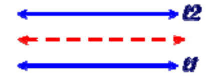
Fixed distance from a line.



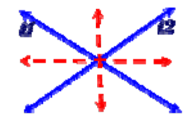
Equidistant from 2 points.



Equidistant 2 parallel lines.



Equidistant from 2 intersecting lines



Triangles:
By Sides:
Scalene – no congruent sides
Isosceles – 2 congruent sides
Equilateral – 3 congruent sides
By Angles:
Acute – all acute angles
Right – one right angle
Obtuse – one obtuse angle
Equiangular – 3 congruent angles(60º)
Equilateral $\leftrightarrow$ Equiangular

Exterior angle of a triangle equals the sum of the 2 non-adjacent interior angles.

Mid-segment of a triangle is parallel to the third side and half the length of the third side.

Polygon Interior/Exterior Angles:
Sum of int. angles = $180(n-2)$

Each int. angle (regular) = $\frac{180(n-2)}{n}$

Sum of ext. angles = 360

Each ext. angle (regular) = $\frac{360}{n}$

Congruent Triangles

SSS
SAS
ASA
AAS
HL (right triangles only)

NO donkey theorem (SSA or ASS)

CPCTC (use after the triangles are congruent)

Related Conditionals:
Converse: switch if and then
Inverse: negate if and then
Contrapositive: inverse of the converse (contrapositive has the same truth value as the original statement)

Inequalities:
--Sum of the lengths of any two sides of a triangle is greater than the length of the third side.
--Longest side of a triangle is opposite the largest angle.
--Exterior angle of a triangle is greater than either of the two non-adjacent interior angles.

Pythagorean Theorem:
$c^2 = a^2 + b^2$
Converse: If the sides of a triangle satisfy $c^2 = a^2 + b^2$ then the triangle is a right triangle.

Similar Triangles:
AA
SSS for similarity
SAS for similarity
Corresponding sides of similar triangles are in proportion.

Mean Proportional in Right Triangle:

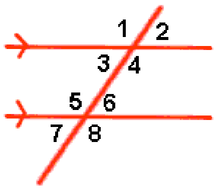
Altitude Rule:

$$\frac{\text{part hyp}}{\text{altitude}} = \frac{\text{altitude}}{\text{other part hyp}}$$

Leg Rule:

$$\frac{\text{hyp}}{\text{leg}} = \frac{\text{leg}}{\text{projection}}$$

Parallels: If lines are parallel …



Corresponding angles are equal.
$m\angle1=m\angle5$, $m\angle2=m\angle6$, $m\angle3=m\angle7$, $m\angle4=m\angle8$
Alternate Interior angles are equal.
$m\angle3=m\angle6$, $m\angle4=m\angle5$
Alternate Exterior angles are equal.
$m\angle1=m\angle8$, $m\angle2=m\angle7$
Same side interior angles are supp.
$m\angle3 + m\angle5=180$, $m\angle4 + m\angle6=180$

Quadrilaterals:

Parallelogram:
opp. sides parallel
opp sides =
opp angles =
consec. angles supp
diag bis each other
Rectangle: add 4 rt angles, diag. =
Rhombus: add 4 = sides, diag. perp, diag bisect angles.
Square: All from above.

Trapezoid:
Only one set parallel sides.

Median of trap is parallel to both bases and = ½ sum bases.
Isosceles Trap:
legs =
base angles =
diagonals =
opp angles supp

Transformations:

$r_{x-axis}(x,y)=(x,-y)$
$r_{y-axis}(x,y)=(-x,y)$
$r_{y=x}(x,y)=(y,x)$
$r_{y=-x}(x,y)=(-y,-x)$
$r_{origin}(x,y)=(-x,-y)$
$T_{a,b}(x,y)=(x+a,y+b)$
$D_k(x,y)=(kx,ky)$
$R_{90^\circ}(x,y)=(-y,x)$
$R_{180^\circ}(x,y)=(-x,-y)$
$R_{270^\circ}(x,y)=(y,-x)$

Glide reflection is composition of a reflection and a translation.

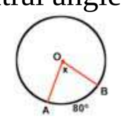
Isometry – keeps length.

Orientation – label order

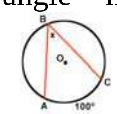
Circle Segments
In a circle, a radius perpendicular to a chord bisects the chord.
Intersecting Chords Rule:
(segment part)•(segment part) = (segment part)•(segment part)
Secant-Secant Rule:
(whole secant)•(external part) = (whole secant)•(external part)
Secant-Tangent Rule:
(whole secant)•(external part) = $(\text{tangent})^2$
Hat Rule: Two tangents are equal.

Circle Angles:

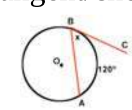
Central angle = arc



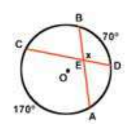
Inscribed angle = half arc



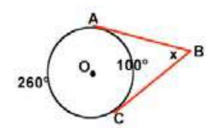
Angle by tangent/chord = half arc

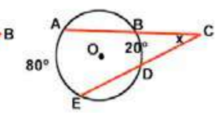


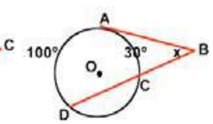
Angle formed by 2 chords = half the sum of arcs



Angle formed by 2 tangents, or 2 secants, or a tangent/secant = half the difference of arcs







Slopes and Equations:

$$m=\frac{vertical\ change}{horizontal\ change}=\frac{y_2-y_1}{x_2-x_1}.$$

$y=mx+b$ slope-intercept

$y-y_1=m(x-x_1)$ point-slope

Coordinate Geometry Formulas:

Distance Formula:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Midpoint Formula:

$$(x,y)=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

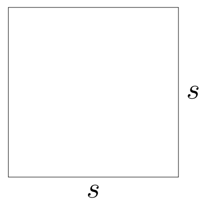
Circles:

Equation of circle center at origin:
$x^2+y^2=r^2$ where r is the radius.
Equation of circle not at origin:
$(x-h)^2+(y-k)^2=r^2$ where (h,k) is the center and r is the radius.

2D GEOMETRY FORMULAS

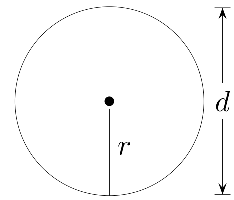
SQUARE

s = side
Area: $A = s^2$
Perimeter: $P = 4s$



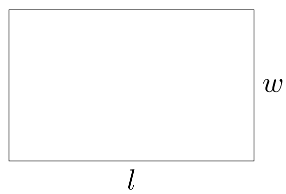
CIRCLE

r = radius, d = diameter
Diameter: $d = 2r$
Area: $A = \pi r^2$
Circumference: $C = 2\pi r = \pi d$



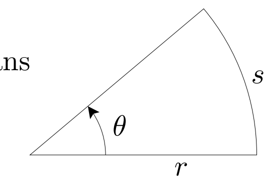
RECTANGLE

l = length, w = width
Area: $A = lw$
Perimeter: $P = 2l + 2w$



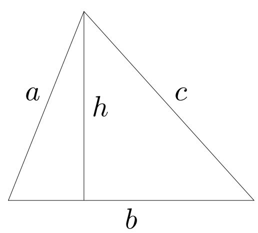
SECTOR OF CIRCLE

r = radius, θ = angle in radians
Area: $A = \frac{1}{2}\theta r^2$
Arc Length: $s = \theta r$



TRIANGLE

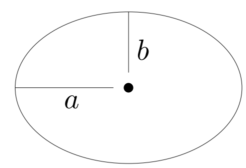
b = base, h = height
Area: $A = \frac{1}{2}bh$
Perimeter: $P = a + b + c$



ELLIPSE

a = semimajor axis
b = semiminor axis
Area: $A = \pi ab$

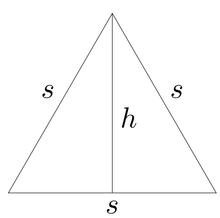
Circumference:
$C \approx \pi\left(3(a+b) - \sqrt{(a+3b)(b+3a)}\right)$



EQUILATERAL TRIANGLE

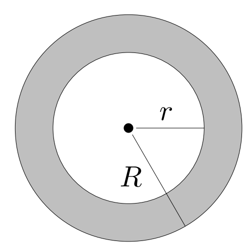
s = side
Height: $h = \frac{\sqrt{3}}{2}s$

Area: $A = \frac{\sqrt{3}}{4}s^2$



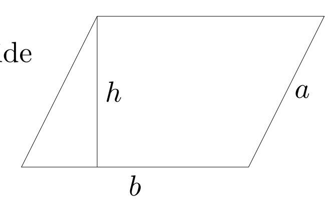
ANNULUS

r = inner radius,
R = outer radius
Average Radius: $\rho = \frac{1}{2}(r + R)$
Width: $w = R - r$
Area: $A = \pi(R^2 - r^2)$
or $A = 2\pi\rho w$



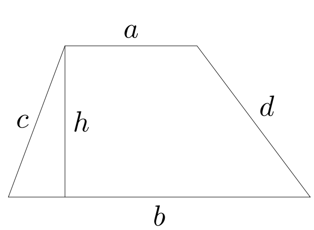
PARALLELOGRAM

b = base, h = height, a = side
Area: $A = bh$
Perimeter: $P = 2a + 2b$



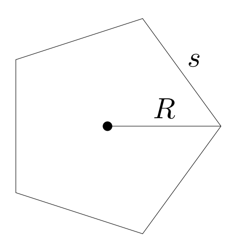
TRAPEZOID

a, b = bases; h = height;
c, d = sides
Area: $A = \frac{1}{2}(a + b)h$
Perimeter:
$P = a + b + c + d$



REGULAR POLYGON

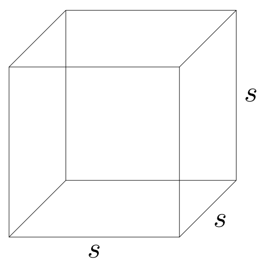
s = side length,
n = number of sides
Circumradius: $R = \frac{1}{2}s\csc(\frac{\pi}{n})$
Area: $A = \frac{1}{4}ns^2\cot(\frac{\pi}{n})$
or $A = \frac{1}{2}nR^2\sin(\frac{2\pi}{n})$



3D GEOMETRY FORMULAS

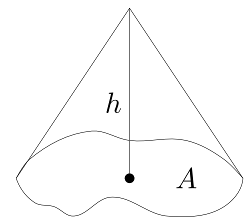
CUBE

s = side
Volume: $V = s^3$
Surface Area: $S = 6s^2$



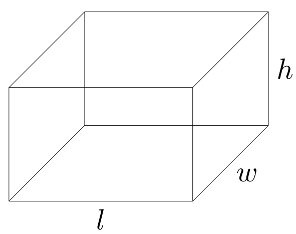
GENERAL CONE OR PYRAMID

A = area of base, h = height
Volume: $V = \frac{1}{3}Ah$



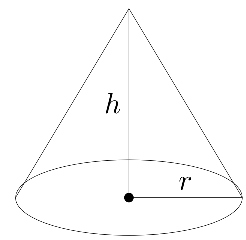
RECTANGULAR SOLID

l = length, w = width,
h = height
Volume: $V = lwh$
Surface Area:
$S = 2lw + 2lh + 2wh$



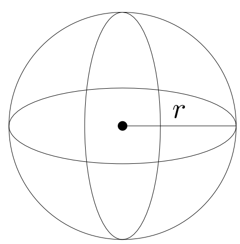
RIGHT CIRCULAR CONE

r = radius, h = height
Volume: $V = \frac{1}{3}\pi r^2 h$
Surface Area:
$S = \pi r\sqrt{r^2 + h^2} + \pi r^2$



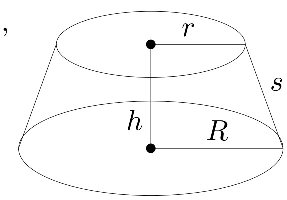
SPHERE

r = radius
Volume: $V = \frac{4}{3}\pi r^3$
Surface Area: $S = 4\pi r^2$



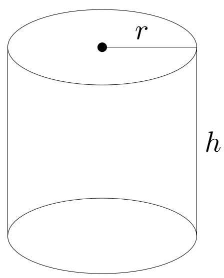
FRUSTUM OF A CONE

r = top radius, R = base radius,
h = height, s = slant height
Volume: $V = \frac{\pi}{3}(r^2 + rR + R^2)h$
Surface Area:
$S = \pi s(R + r) + \pi r^2 + \pi R^2$



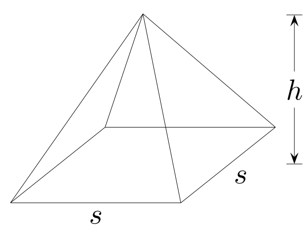
RIGHT CIRCULAR CYLINDER

r = radius, h = height
Volume: $V = \pi r^2 h$
Surface Area: $S = 2\pi rh + 2\pi r^2$



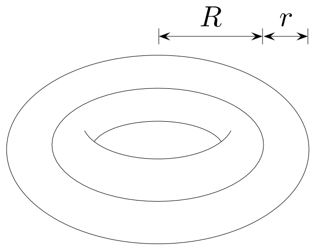
SQUARE PYRAMID

s = side, h = height
Volume: $V = \frac{1}{3}s^2 h$
Surface Area:
$S = s(s + \sqrt{s^2 + 4h^2})$



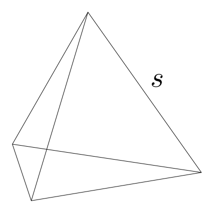
TORUS

r = tube radius,
R = torus radius
Volume: $V = 2\pi^2 r^2 R$
Surface Area: $S = 4\pi^2 rR$



REGULAR TETRAHEDRON

s = side
Volume: $V = \frac{1}{12}\sqrt{2}s^3$
Surface Area: $S = \sqrt{3}s^2$



Analytic Geometry Formulas

1. Lines in two dimensions

Line forms

Slope - intercept form:

$$y = mx + b$$

Two point form:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Point slope form:

$$y - y_1 = m(x - x_1)$$

Intercept form

$$\frac{x}{a} + \frac{y}{b} = 1 \ (a, b \neq 0)$$

Normal form:

$$x \cdot \cos\sigma + y \sin\sigma = p$$

Parametric form:

$$x = x_1 + t\cos\alpha$$

$$y = y_1 + t\sin\alpha$$

Point direction form:

$$\frac{x - x_1}{A} = \frac{y - y_1}{B}$$

where (A,B) is the direction of the line and $P_1(x_1, y_1)$ lies on the line.

General form:

$$A \cdot x + B \cdot y + C = 0 \quad A \neq 0 \ \ or \ B \neq 0$$

Distance

The distance from $Ax + By + C = 0$ to $P_1(x_1, y_1)$ is

$$d = \frac{|A \cdot x_1 + B \cdot y_1 + C|}{\sqrt{A^2 + B^2}}$$

Concurrent lines

Three lines

$$A_1 x + B_1 y + C_1 = 0$$

$$A_2 x + B_2 y + C_2 = 0$$

$$A_3 x + B_3 y + C_3 = 0$$

are concurrent if and only if:

$$\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = 0$$

Line segment

A line segment P_1P_2 can be represented in parametric form by

$$x = x_1 + (x_2 - x_1)t$$

$$y = y_1 + (y_2 - y_1)t$$

$$0 \le t \le 1$$

Two line segments P_1P_2 and P_3P_4 intersect if any only if the numbers

$$s = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}}{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_4 & y_3 - y_4 \end{vmatrix}} \quad and \quad t = \frac{\begin{vmatrix} x_3 - x_1 & y_3 - y_1 \\ x_3 - x_4 & y_3 - y_4 \end{vmatrix}}{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_4 & y_3 - y_4 \end{vmatrix}}$$

satisfy $0 \le s \le 1 \ and \ 0 \le t \le 1$

2. Triangles in two dimensions

Area

The area of the triangle formed by the three lines:

$$A_1 x + B_1 y + C_1 = 0$$

$$A_2 x + B_2 y + C_2 = 0$$

$$A_3 x + B_3 y + C_3 = 0$$

is given by

$$K = \frac{\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}^2}{2 \cdot \begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix} \cdot \begin{vmatrix} A_2 & B_2 \\ A_3 & B_3 \end{vmatrix} \cdot \begin{vmatrix} A_3 & B_3 \\ A_1 & B_1 \end{vmatrix}}$$

The area of a triangle whose vertices are $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$:

$$K = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$K = \frac{1}{2}\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix}.$$

Centroid

The centroid of a triangle whose vertices are $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$:

$$(x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

Incenter

The incenter of a triangle whose vertices are $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$:

$$(x, y) = \left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$$

where a is the length of P_2P_3, b is the length of P_1P_3, and c is the length of P_1P_2.

Circumcenter

The circumcenter of a triangle whose vertices are $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$:

$$(x, y) = \left(\frac{\begin{vmatrix} x_1^2 + y_1^2 & y_1 & 1 \\ x_2^2 + y_2^2 & y_2 & 1 \\ x_3^2 + y_3^2 & y_3 & 1 \end{vmatrix}}{2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}, \frac{\begin{vmatrix} x_1 & x_1^2 + y_1^2 & 1 \\ x_2 & x_2^2 + y_2^2 & 1 \\ x_3 & x_3^2 + y_3^2 & 1 \end{vmatrix}}{2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}} \right)$$

Orthocenter

The orthocenter of a triangle whose vertices are $P_1(x_1, y_1)$, $P_2(x_2, y_2)$ and $P_3(x_3, y_3)$:

$$(x, y) = \left(\frac{\begin{vmatrix} y_1 & x_2x_3 + y_1^2 & 1 \\ y_2 & x_3x_1 + y_2^2 & 1 \\ y_3 & x_1x_2 + y_3^2 & 1 \end{vmatrix}}{2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}}, \frac{\begin{vmatrix} x_1^2 + y_2y_3 & x_1 & 1 \\ x_2^2 + y_3y_1 & x_2 & 1 \\ x_3^2 + y_1y_2 & x_3 & 1 \end{vmatrix}}{2\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}} \right)$$

3. Circle

Equation of a circle

In an x-y coordinate system, the circle with centre (a, b) and radius r is the set of all points (x, y) such that:

$$(x-a)^2 + (y-b)^2 = r^2$$

Circle is centred at the origin

$$x^2 + y^2 = r^2$$

Parametric equations

$$x = a + r\cos t$$

$$y = b + r\sin t$$

where t is a parametric variable.

In polar coordinates the equation of a circle is:

$$r^2 - 2rr_o \cos(\theta - \varphi) + r_o^2 = a^2$$

Area

$$A = r^2\pi$$

Circumference

$$c = \pi \cdot d = 2\pi \cdot r$$

Theoremes:

(Chord theorem)

The chord theorem states that if two chords, CD and EF, intersect at G, then:

$$CD \cdot DG = EG \cdot FG$$

(Tangent-secant theorem)

If a tangent from an external point D meets the circle at C and a secant from the external point D meets the circle at G and E respectively, then

$$DC^2 = DG \cdot DE$$

(Secant - secant theorem)

If two secants, DG and DE, also cut the circle at H and F respectively, then:

$$DH \cdot DG = DF \cdot DE$$

(Tangent chord property)

The angle between a tangent and chord is equal to the subtended angle on the opposite side of the chord.

4. Conic Sections

The Parabola

The set of all points in the plane whose distances from a fixed point, called the focus, and a fixed line, called the directrix, are always equal.

The standard formula of a parabola:

$$y^2 = 2px$$

Parametric equations of the parabola:

$$x = 2pt^2$$
$$y = 2pt$$

Tangent line

Tangent line in a point $D(x_0, y_0)$ of a parabola $y^2 = 2px$

$$y_0 y = p\left(x + x_0\right)$$

Tangent line with a given slope (m)

$$y = mx + \frac{p}{2m}$$

Tangent lines from a given point

Take a fixed point $P(x_0, y_0)$.The equations of the tangent lines are

$$y - y_0 = m_1\left(x - x_0\right) \textit{ and}$$
$$y - y_0 = m_2\left(x - x_0\right) \textit{where}$$
$$m_1 = \frac{y_0 + \sqrt{{y_0}^2 - 2px_0}}{2x_0} \textit{ and}$$
$$m_1 = \frac{y_0 - \sqrt{{y_0}^2 - 2px_0}}{2x_0}$$

The Ellipse

The set of all points in the plane, the sum of whose distances from two fixed points, called the foci, is a constant.

The standard formula of a ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Parametric equations of the ellipse

$$x = a\cos t$$
$$y = b\sin t$$

Tangent line in a point $D(x_0, y_0)$ of a ellipse:

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1$$

Eccentricity:

$$e = \frac{\sqrt{a^2 - b^2}}{a}$$

Foci:

$$\textit{if } a > b \Rightarrow F_1(-\sqrt{a^2 - b^2}, 0)\ \ F_2(\sqrt{a^2 - b^2}, 0)$$
$$\textit{if } a < b \Rightarrow F_1(0, -\sqrt{b^2 - a^2})\ \ F_2(0, \sqrt{b^2 - a^2})$$

Area:

$$K = \pi \cdot a \cdot b$$

The Hyperbola

The set of all points in the plane, the difference of whose distances from two fixed points, called the foci, remains constant.

The standard formula of a hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Parametric equations of the Hyperbola

$$x = \frac{a}{\sin t}$$
$$y = \frac{b\sin t}{\cos t}$$

Tangent line in a point $D(x_0, y_0)$ of a hyperbola:

$$\frac{x_0 x}{a^2} - \frac{y_0 y}{b^2} = 1$$

Foci:

$$\textit{if } a > b \Rightarrow F_1(-\sqrt{a^2 + b^2}, 0)\ \ F_2(\sqrt{a^2 + b^2}, 0)$$
$$\textit{if } a < b \Rightarrow F_1(0, -\sqrt{b^2 + a^2})\ \ F_2(0, \sqrt{b^2 + a^2})$$

Asymptotes:

$$\textit{if } a > b \Rightarrow y = \frac{b}{a}x \textit{ and } y = -\frac{b}{a}x$$
$$\textit{if } a < b \Rightarrow y = \frac{a}{b}x \textit{ and } y = -\frac{a}{b}x$$

5. Planes in three dimensions

Plane forms

Point direction form:

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

where P1(x1,y1,z1) lies in the plane, and the direction (a,b,c) is normal to the plane.

General form:

$$Ax+By+Cz+D=0$$

where direction (A,B,C) is normal to the plane.

Intercept form:

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

this plane passes through the points (a,0,0), (0,b,0), and (0,0,c).

Three point form

$$\begin{vmatrix} x-x_3 & y-y_3 & z-z_3 \\ x_1-x_3 & y_1-y_3 & z_1-z_3 \\ x_2-x_3 & y_2-y_3 & z_2-z_3 \end{vmatrix}=0$$

Normal form:

$$x\cos\alpha+y\cos\beta+z\cos\gamma=p$$

Parametric form:

$$x=x_1+a_1s+a_2t$$
$$y=y_1+b_1s+b_2t$$
$$z=z_1+c_1s+c_2t$$

where the directions (a1,b1,c1) and (a2,b2,c2) are parallel to the plane.

Angle between two planes:

The angle between two planes:

$$A_1x+B_1y+C_1z+D_1=0$$
$$A_2x+B_2y+C_2z+D_2=0$$

is

$$\arccos\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}$$

The planes are parallel if and only if

$$\frac{A_1}{A_2}=\frac{B_1}{B_2}=\frac{C_1}{C_2}$$

The planes are perpendicular if and only if

$$A_1A_2+B_1B_2+C_1C_2=0$$

Equation of a plane

The equation of a plane through $P_1(x_1,y_1,z_1)$ and parallel to directions (a_1,b_1,c_1) and (a_2,b_2,c_2) has equation

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}=0$$

The equation of a plane through $P_1(x_1,y_1,z_1)$ and $P_2(x_2,y_2,z_2)$, and parallel to direction (a,b,c), has equation

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a & b & c \end{vmatrix}=0$$

Distance

The distance of P1(x1,y1,z1) from the plane Ax + By + Cz + D = 0 is

$$d=\frac{Ax_1+By_1+Cz_1}{\sqrt{A^2+B^2+C^2}}$$

Intersection

The intersection of two planes

$$A_1x+B_1y+C_1z+D_1=0,$$
$$A_2x+B_2y+C_2z+D_2=0,$$

is the line

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c},$$

where

$$a=\begin{vmatrix} B_1 & C_1 \\ B_2 & C_2 \end{vmatrix}$$

$$b=\begin{vmatrix} C_1 & A_1 \\ C_2 & A_2 \end{vmatrix}$$

$$c=\begin{vmatrix} A_1 & B_1 \\ A_2 & B_2 \end{vmatrix}$$

$$x_1=\frac{b\begin{vmatrix} D_1 & C_1 \\ D_2 & C_2 \end{vmatrix}-c\begin{vmatrix} D_1 & B_1 \\ D_2 & B_2 \end{vmatrix}}{a^2+b^2+c^2}$$

$$y_1=\frac{c\begin{vmatrix} D_1 & A_1 \\ D_2 & A_2 \end{vmatrix}-c\begin{vmatrix} D_1 & C_1 \\ D_2 & C_2 \end{vmatrix}}{a^2+b^2+c^2}$$

$$z_1=\frac{a\begin{vmatrix} D_1 & B_1 \\ D_2 & B_2 \end{vmatrix}-b\begin{vmatrix} D_1 & A_1 \\ D_2 & A_2 \end{vmatrix}}{a^2+b^2+c^2}$$

If a = b = c = 0, then the planes are parallel.

TRIGONOMETRY FORMULAS

$$\cos^2(x)+\sin^2(x)=1 \qquad 1+\tan^2(x)=\sec^2(x) \qquad \cot^2(x)+1=\csc^2(x)$$

$$\cos(x\pm y)=\cos(x)\cos(y)\mp\sin(x)\sin(y)$$
$$\sin(x\pm y)=\sin(x)\cos(y)\pm\cos(x)\sin(y)$$
$$\tan(x\pm y)=\frac{\tan(x)\pm\tan(y)}{1\mp\tan(x)\tan(y)}$$

$$\sin(2x)=2\sin(x)\cos(x)$$
$$\cos(2x)=\begin{cases}\cos^2(x)-\sin^2(x)\\ 2\cos^2(x)-1\\ 1-2\sin^2(x)\end{cases}$$
$$\tan(2x)=\frac{2\tan(x)}{1-\tan^2(x)}$$

$$c^2=a^2+b^2-2ab\cos(C)$$
$$\frac{\sin(A)}{a}=\frac{\sin(B)}{b}=\frac{\sin(C)}{c}$$

$$\sin^2(x)=\frac{1-\cos(2x)}{2}$$
$$\cos^2(x)=\frac{1+\cos(2x)}{2}$$
$$\tan^2(x)=\frac{1-\cos(2x)}{1+\cos(2x)}$$

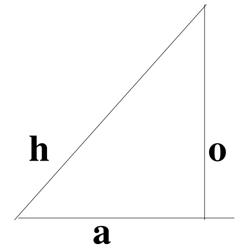
$$\cos\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1+\cos(x)}{2}}$$
$$\sin\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1-\cos(x)}{2}}$$
$$\tan\left(\frac{x}{2}\right)=\pm\sqrt{\frac{1-\cos(x)}{1+\cos(x)}}$$

$$\sin(x)\sin(y)=\tfrac{1}{2}[\cos(x-y)-\cos(x+y)]$$
$$\cos(x)\cos(y)=\tfrac{1}{2}[\cos(x-y)+\cos(x+y)]$$
$$\sin(x)\cos(y)=\tfrac{1}{2}[\sin(x+y)+\sin(x-y)]$$
$$\cos(x)\sin(y)=\tfrac{1}{2}[\sin(x+y)-\sin(x-y)]$$

$$\sin(x)+\sin(y)=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin(x)-\sin(y)=2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$$
$$\cos(x)+\cos(y)=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos(x)-\cos(y)=-2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

For two vectors **A** and **B, A·B** = ||**A**||||**B**||cos(θ)

The well known results: soh, cah, toa



soh: s stands for sine, o stands for opposite and h stands for hypotenuse, $\sin x = \frac{o}{h}$

cah: c stands for cosine, a stands for adjacent h stands for hypotenuse, $\cos x = \frac{a}{h}$

toa: t stands for tan, o stands for opposite and a stands for adjacent, $\tan x = \frac{o}{a}$

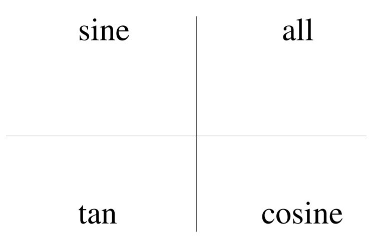
Where x is the angle between the hypotenuse and the adjacent.
Other three trigonometric functions have the following relations:

$\csc x = \frac{1}{\sin x} = \frac{h}{o}$, $\sec x = \frac{1}{\cos x} = \frac{h}{a}$ and $\cot x = \frac{1}{\tan x} = \frac{a}{o}$

Important values:

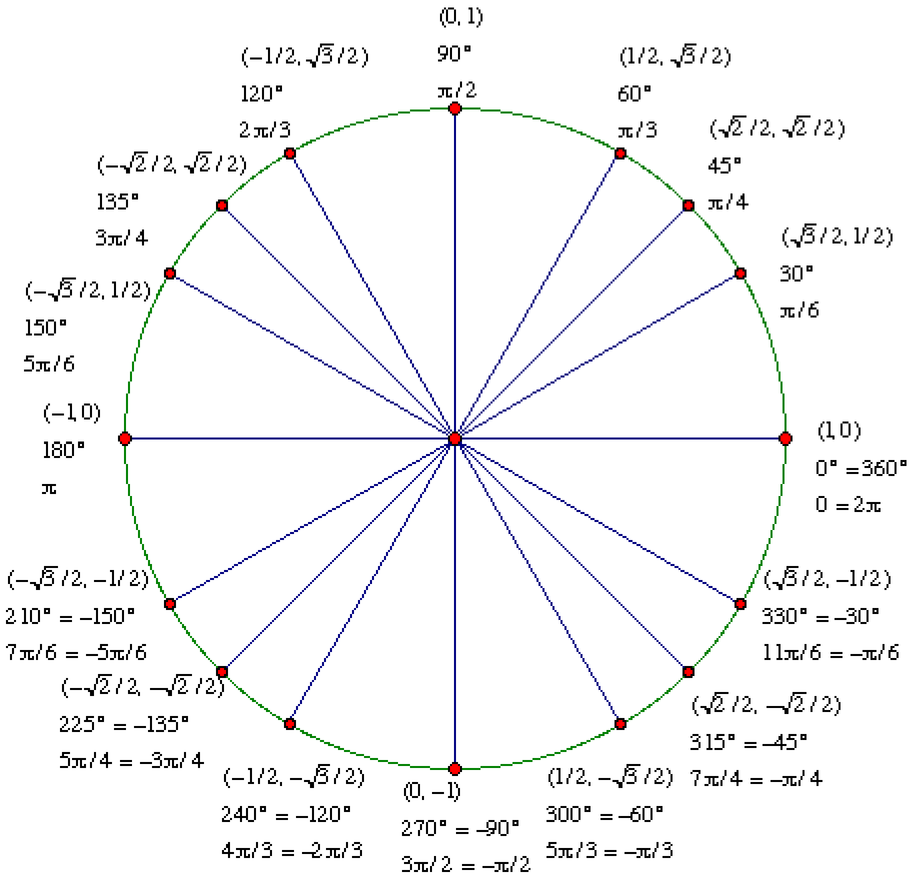
	0	$30^0 = \frac{\pi}{6}$	$45^0 = \frac{\pi}{4}$	$60^0 = \frac{\pi}{3}$	$90^0 = \frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
csc	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	undefined
cot	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

$\sin(n\pi \pm x) = [\,?\,]\sin x$, $\cos(n\pi \pm x) = [\,?\,]\cos x$, $\tan(n\pi \pm x) = [\,?\,]\tan x$ **,** the sign **?** is for plus or minus depending on the position of the terminal side. One may remember the four-quadrant rule: (**A**ll **S**tudents **T**ake **C**alculus: A = all, S = sine, T = tan, C = cosine)



Example: Find the value of $\sin 300^0$. We may write $\sin 300^0 = \sin(2 \cdot 180^0 - 60^0) = [-]\sin 60^0 = -\frac{\sqrt{3}}{2}$, in this case the terminal side is in quadrant four where sine is negative.

In the following diagram, each point on the unit circle is labeled first with its coordinates (exact values), then with the angle in degrees, then with the angle in radians. Points in the lower hemisphere have both positive and negative angles marked.



7. TRIGONOMETRIC FUNCTIONS

Synopsis :

1. Let $\theta \in R$. Take an angle of measure θ radians in the standard position. Let P(x, y) be a point on the terminal side of the angle θ such that OP = r(> 0). Then

   i) $\frac{y}{r}$ is called sine of θ and it is denoted by $\sin\theta$.

   ii) $\frac{x}{r}$ is called cosine of θ and it is denoted by $\cos\theta$

   iii) $\frac{y}{x}$ $(x \neq 0)$ is called tangent of θ and it is denoted by $\tan\theta$.

   iv) $\frac{x}{y}$ $(y \neq 0)$ is called cotangent of θ and it is denoted by $\cot\theta$.

   v) $\frac{r}{x}$ $(x \neq 0)$ is called secant of θ and it is denoted by $\sec\theta$.

   vi) $\frac{r}{y}$ $(y \neq 0)$ is called cosecant of θ and it is denoted by $\mathrm{cosec}\,\theta$.

   These six functions (ratios) are called trigonometric functions (ratios).

2. $\sin\theta.\mathrm{cosec}\,\theta = 1$, $\sin\theta = \frac{1}{\mathrm{cosec}\,\theta}$, $\mathrm{cosec}\,\theta = \frac{1}{\sin\theta}$

3. $\cos\theta.\sec\theta = 1$, $\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta = \frac{1}{\cos\theta}$

4. $\tan\theta.\cot\theta = 1$, $\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\tan\theta}$

5. $\frac{\sin\theta}{\cos\theta} = \tan\theta$, $\frac{\cos\theta}{\sin\theta} = \cot\theta$

6. $\sin^2\theta + \cos^2\theta = 1$,

   $\sin^2\theta = 1 - \cos^2\theta$,

   $\cos^2\theta = 1 - \sin^2\theta$

7. $1+\tan^2\theta = \sec^2\theta$, $\tan^2\theta = \sec^2\theta - 1$, $\sec^2\theta - \tan^2\theta = 1$.

8. $1 + \cot^2\theta = \mathrm{cosec}^2\theta$, $\cot^2\theta = \mathrm{cosec}^2\theta - 1$,

   $\mathrm{cosec}^2\theta - \cot^2\theta = 1$.

9. $\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$.

10. $\operatorname{cosec}\theta + \cot\theta = \dfrac{1}{\operatorname{cosec}\theta - \cot\theta}$.

11. The values of the trigonometric functions of some standard angles :

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$	2π
sin	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	–1	0
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0	–1	0	1

12. Trigonometric functions of $2n\pi + \theta$; $n\in Z$

 1) $\sin(2n\pi + \theta) = \sin\theta$, $\cos(2n\pi + \theta) = \cos\theta$

 2) $\tan(2n\pi + \theta) = \tan\theta$, $\cot(2n\pi + \theta) = \cot\theta$

 3) $\sec(2n\pi + \theta) = \sec\theta$, $\operatorname{cosec}(2n\pi + \theta) = \operatorname{cosec}\theta$

13. Trigonometric functions of $(-\theta)$, for all values of θ

 1) $\sin(-\theta) = -\sin\theta$, 2) $\cos(-\theta) = \cos\theta$,

 3) $\tan(-\theta) = -\tan\theta$, 4) $\cot(-\theta) = -\cot\theta$,

 5) $\sec(-\theta) = \sec\theta$, 6) $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

14. The values of trigonometric functions of any angle can be represented in terms of an angle in the first quadrant

 Let $A = n.\frac{\pi}{2} \pm \theta$ where $n\in Z$, $0 \le \theta \le \frac{\pi}{2}$. Then

 i) $\sin\left(n.\frac{\pi}{2} \pm \theta\right) = \pm\sin\theta$, if n is even

 $= \pm\cos\theta$, if n is odd

 ii) $\cos\left(n.\frac{\pi}{2} \pm \theta\right) = \pm\cos\theta$, if n is even

 $= \pm\sin\theta$, if n is odd

 iii) $\tan\left(n.\frac{\pi}{2} \pm \theta\right) = \pm\tan\theta$, if n is even

 $= \pm\cot\theta$, if n is odd

 iv) $\cot\left(n.\frac{\pi}{2} \pm \theta\right) = \pm\cot\theta$, if n is even

 $= \pm\tan\theta$, if n is odd

 v) $\sec\left(n.\frac{\pi}{2} \pm \theta\right) = \pm\sec\theta$, if n is even

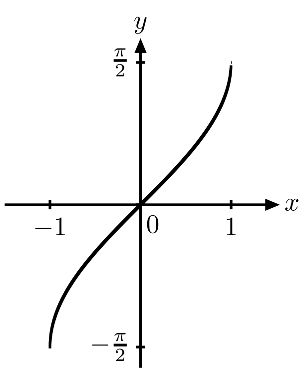
 $= \pm\operatorname{cosec}\theta$, if n is odd

 vi) $\operatorname{cosec}\left(n.\frac{\pi}{2} \pm \theta\right) = \pm\operatorname{cosec}\theta$, if n is even

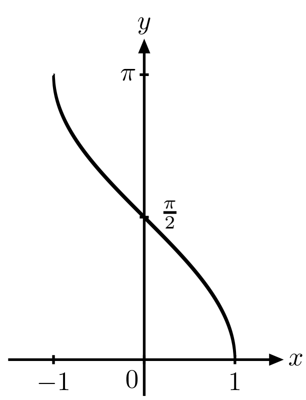
 $= \pm\sec\theta$, if n is odd

Inverse Trigonometric Functions

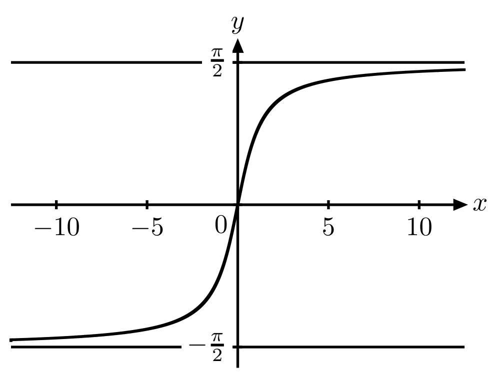
Function	Domain	Range
$\arcsin x$	$[-1, 1]$	$\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$
$\arccos x$	$[-1, 1]$	$[0, \pi]$
$\arctan x$	$(-\infty, \infty)$	$\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$



Graph of $y = \arcsin x$



Graph of $y = \arccos x$



Graph of $y = \arctan x$

Composition

$$\sin(\arcsin x) = x, \quad \text{for all } x \in [-1, 1] \qquad \arcsin(\sin x) = x, \quad \text{for all } x \in \left[-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right]$$
$$\cos(\arccos x) = x, \quad \text{for all } x \in [-1, 1] \qquad \arccos(\cos x) = x, \quad \text{for all } x \in [0, \pi]$$
$$\tan(\arctan x) = x, \quad \text{for all } x \in (-\infty, \infty) \qquad \arctan(\tan x) = x, \quad \text{for all } x \in \left(-\tfrac{\pi}{2}, \tfrac{\pi}{2}\right)$$

Derivative

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

Notation

$$\arcsin x = \sin^{-1} x \qquad \arccos x = \cos^{-1} x \qquad \arctan x = \tan^{-1} x$$

Note that the -1 in these notations is not an exponent, e.g., $\sin^{-1} x \neq \frac{1}{\sin x}$.

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   iv) $\frac{x}{y}$ $(y \neq 0)$ is called cotangent of θ and it is denoted by $\cot\theta$.

   v) $\frac{r}{x}$ $(x \neq 0)$ is called secant of θ and it is denoted by $\sec\theta$.

   vi) $\frac{r}{y}$ $(y \neq 0)$ is called cosecant of θ and it is denoted by $\operatorname{cosec}\theta$.

   These six functions (ratios) are called trigonometric functions (ratios).

2. $\sin\theta.\operatorname{cosec}\theta = 1$, $\sin\theta = \frac{1}{\operatorname{cosec}\theta}$, $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

3. $\cos\theta.\sec\theta = 1$, $\cos\theta = \frac{1}{\sec\theta}$, $\sec\theta = \frac{1}{\cos\theta}$

4. $\tan\theta.\cot\theta = 1$, $\tan\theta = \frac{1}{\cot\theta}$, $\cot\theta = \frac{1}{\tan\theta}$

5. $\frac{\sin\theta}{\cos\theta} = \tan\theta$, $\frac{\cos\theta}{\sin\theta} = \cot\theta$

6. $\sin^2\theta + \cos^2\theta = 1$,

   $\sin^2\theta = 1 - \cos^2\theta$,

   $\cos^2\theta = 1 - \sin^2\theta$

7. $1+\tan^2\theta = \sec^2\theta$, $\tan^2\theta = \sec^2\theta - 1$, $\sec^2\theta - \tan^2\theta = 1$.

8. $1 + \cot^2\theta = \operatorname{cosec}^2\theta$, $\cot^2\theta = \operatorname{cosec}^2\theta - 1$,

   $\operatorname{cosec}^2\theta - \cot^2\theta = 1$.

9. $\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$.

10. cosec θ + cot θ = $\frac{1}{\operatorname{cosec}\theta - \cot\theta}$.

11. The values of the trigonometric functions of some standard angles :

θ	0	π/6	π/4	π/3	π/2	π	3π/2	2π
sin	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1	0	–1	0
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0	–1	0	1

12. Trigonometric functions of 2nπ + θ; n∈Z

 1) sin(2nπ + θ) = sinθ, cos(2nπ + θ) = cosθ

 2) tan(2nπ + θ) = tanθ, cot(2nπ + θ) = cotθ

 3) sec(2nπ + θ) = secθ, cosec(2nπ + θ) = cosecθ

13. Trigonometric functions of (–θ), for all values of θ

 1) sin(–θ) = –sin θ, 2) cos(–θ) = cos θ,

 3) tan(–θ) = –tan θ, 4) cot(–θ) = –cot θ,

 5) sec(–θ) = sec θ, 6) cosec(–θ) = –cosec θ

14. The values of trigonometric functions of any angle can be represented in terms of an angle in the first quadrant

 Let A = $n.\frac{\pi}{2} \pm \theta$ where n∈Z, $0 \le \theta \le \frac{\pi}{2}$. Then

 i) $\sin\left(n.\frac{\pi}{2} \pm \theta\right)$ = ± sin θ, if n is even

 = ± cos θ, if n is odd

 ii) $\cos\left(n.\frac{\pi}{2} \pm \theta\right)$ = ± cos θ, if n is even

 = ± sin θ, if n is odd

 iii) $\tan\left(n.\frac{\pi}{2} \pm \theta\right)$ = ± tan θ, if n is even

 = ± cot θ, if n is odd

 iv) $\cot\left(n.\frac{\pi}{2} \pm \theta\right)$ = ± cot θ, if n is even

 = ± tan θ, if n is odd

 v) $\sec\left(n.\frac{\pi}{2} \pm \theta\right)$ = ± sec θ, if n is even

 = ± cosec θ, if n is odd

 vi) $\operatorname{cosec}\left(n.\frac{\pi}{2} \pm \theta\right)$ = ± cosec θ, if n is even

 = ± sec θ, if n is odd

PROPERTIES OF TRIANGLES

1. The perpendicular bisectors of the sides of a triangle are concurrent. The point of concurrence is called circumcentre of the triangle. If S is the circumcentre of ΔABC, then SA = SB = SC. The circle with center S and radius SA passes through the three vertices A, B, C of the triangle. This circle is called circumcircle of the triangle. The radius of the circumcircle of ΔABC is called circumradius and it is denoted by R.

2. Sine Rule : $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R.$

   $\therefore$ a = 2R sin A, b = 2R sin B, c = 2R sin C.

3. Cosine Rule : $a^2 = b^2 + c^2 - 2bc \cos A$, $b^2 = c^2 + a^2 - 2ca \cos B$, $c^2 = a^2 + b^2 - 2ab \cos C$.

4. $\cos A = \frac{b^2+c^2-a^2}{2bc}$, $\cos B = \frac{c^2+a^2-b^2}{2ca}$,

   $\cos C = \frac{a^2+b^2-c^2}{2ab}$.

5. Projection Rule : a = b cos C + c cos B, b = c cos A + a cos C, c = a cos B + b cos A.

6. Tangent Rule or Napier's Analogy : $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2}$,

   $\tan\left(\frac{C-A}{2}\right) = \frac{c-a}{c+a}\cot\frac{B}{2}$,

   $\tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$.

7. Mollweide Rule :

   $$\frac{a+b}{c} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}}, \quad \frac{a-b}{c} = \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\frac{C}{2}}$$

8. $\sin\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$, $\sin\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$, $\sin\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$.

9. $\cos\frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$, $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$, $\cos\frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$.

10. $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$, $\tan\frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$, $\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

11. $\tan\frac{A}{2} = \frac{\Delta}{s(s-a)} = \frac{(s-b)(s-c)}{\Delta}$,

$\tan\frac{B}{2} = \frac{\Delta}{s(s-b)} = \frac{(s-c)(s-a)}{\Delta}$,

$\tan\frac{C}{2} = \frac{\Delta}{s(s-c)} = \frac{(s-a)(s-b)}{\Delta}$.

12. $\cot\frac{A}{2} = \frac{s(s-a)}{\Delta}$, $\cot\frac{B}{2} = \frac{s(s-b)}{\Delta}$, $\cot\frac{C}{2} = \frac{s(s-c)}{\Delta}$

13. Area of ΔABC is $\Delta = \frac{1}{2}bc\sin A = \frac{1}{2}ca\sin B = \frac{1}{2}\sin C = 2R^2 \sin A \sin B \sin C =$

$\frac{abc}{4R}\sqrt{s(s-a)(s-b)(s-c)}$.

14. $r = \frac{\Delta}{s} = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2} = \frac{a}{\cot\frac{B}{2}+\cot\frac{C}{2}} = \frac{b}{\cot\frac{C}{2}+\cot\frac{A}{2}}$

$= \frac{C}{\cot\frac{A}{2}+\cot\frac{B}{2}}$

15. $r_1 = \frac{\Delta}{s-a} = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} = s\tan\frac{A}{2} = (s-b)\cot\frac{C}{2} = (s-c)\cot\frac{B}{2} = \frac{a}{\tan\frac{B}{2}+\tan\frac{C}{2}}$.

16. $r_2 = \frac{\Delta}{s-b} = s\tan\frac{B}{2} = (s-c)\cot\frac{A}{2} = (s-a)\cot\frac{C}{2} =$

$4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2} = \frac{b}{\tan\frac{C}{2}+\tan\frac{A}{2}}$.

17. $r_3 = \frac{\Delta}{s-c} = s\tan\frac{C}{2} = (s-a)\cot\frac{B}{2} = (s-b)\cot\frac{A}{2} =$

$\frac{c}{\tan\frac{A}{2}+\tan\frac{B}{2}}$.

18. $\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3} = \frac{1}{r}$.

19. $r\, r_1 r_2 r_3 = \Delta^2$.

20. i) $\sum a^3 \sin(B-C) = 0$.

ii) $\sum a^3 \cos(B-C) = 3abc$

iii) $a^2 \sin 2B + b^2 \sin 2A = 4\Delta$

21. i) $\cot A + \cot B + \cot C = \dfrac{a^2+b^2+c^2}{4\Delta}$

ii) $\cot\dfrac{A}{2}\cot\dfrac{B}{2}\cot\dfrac{C}{2} = \dfrac{(a+b+c)^2}{4\Delta}$.

22. i) If $a \cos B = b \cos A$, then the triangle is isosceles.

ii) If $a \cos A = b \cos B$, then the triangle is isosceles or right angled.

iii) If $a^2 + b^2 + c^2 = 8R^2$, then the triangle is right angled.

iv) If $\cos^2 A + \cos^2 B + \cos^2 C = 1$, then the triangle is right angled.

v) If $\cos A = \dfrac{\sin B}{2 \sin C}$, then the triangle is isosceles.

vi) If $\dfrac{a}{\cos A} = \dfrac{b}{\cos B} = \dfrac{c}{\cos C}$, then the triangle is equilateral.

vii) If $\cos A + \cos B + \cos C = 3/2$, then the triangle is equilateral.

viii) If $\sin A + \sin B + \sin C = \dfrac{3\sqrt{3}}{2}$, then the triangle is equilateral.

ix) If $\cot A + \cot B + \cot C = \sqrt{3}$, then the triangle is equilateral.

23. i) If $\dfrac{a^2+b^2}{a^2-b^2} = \dfrac{\sin(A+B)}{\sin(A-B)}$, then $C = 90°$.

ii) If $\dfrac{a+b}{b+c} + \dfrac{b}{c+a} = 1$, then $C = 60°$.

iii) If $\dfrac{1}{a+b} + \dfrac{1}{a+c} = \dfrac{3}{a+b+c}$, then $A = 60°$

iv) If $\dfrac{b}{a^2-c^2} + \dfrac{c}{a^2-b^2} = 0$, then $A = 60°$.

i) a, b, c are In H.P. $\Leftrightarrow \sin^2\dfrac{A}{2}, \sin^2\dfrac{B}{2}, \sin^2\dfrac{C}{2}$ are in H.P.

ii) a, b, c are in A.P. $\Leftrightarrow \cot\dfrac{A}{2}, \cot\dfrac{B}{2}, \cot\dfrac{C}{2}$ are in A.P.

iii) a, b, c are in A.P. $\Leftrightarrow \tan\dfrac{A}{2}, \tan\dfrac{B}{2}, \tan\dfrac{C}{2}$ are in H.P.

iv) a^2, b^2, c^2 are in A.P. $\Leftrightarrow$ cotA, cotB, cotC are in A.P.

v) a^2, b^2, c^2 are in A.P. $\Leftrightarrow$ tanA, tanB, tanC are in H.P

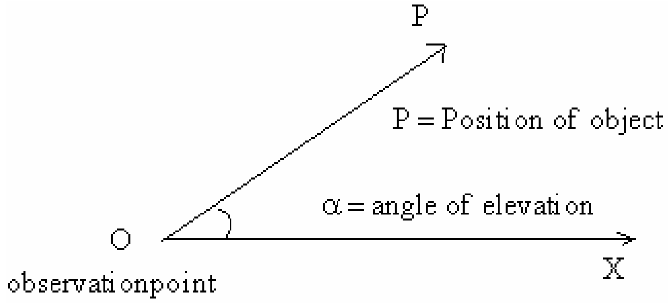
@MCERS

HEIGHTS AND DISTANCES

Synopsis :

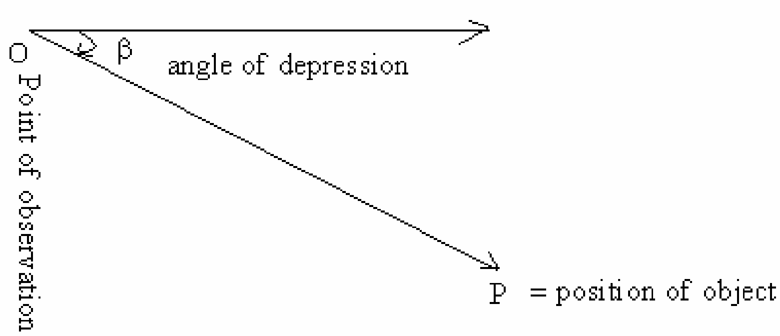
ANGLE OF ELEVATION

1. If the position of the object is above the position of the observation then the angle made by the line joining object and observation point with the horizontal line drawn at the observation point is called angle of elevation.



ANGLE OF DEPRESSION:

2. If the position of the object is below the position of the observation the angle made by the line joining object and observation point with the horizontal line drawn at the observation point is called angle of depression.



3. a. The angle of elevation of the top of a tower, standing on a horizontal plane, from a point A is . After walking a distance 'd' metres towards the foot of the tower, the angle of elevation is found to be β .

The height of the tower h = $\dfrac{d\sin\beta\sin\alpha}{Sin(\beta-\alpha)}$

(or) $h = \dfrac{d}{Cot\alpha - \cot\beta}$

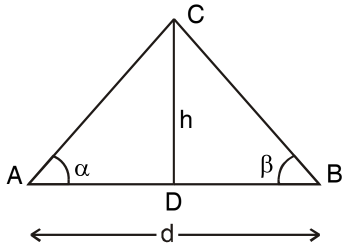
Where $\overline{AB} = \text{d}$

4. If the Points of observation A and B lie on either side of the tower, then height of the tower

h = $\dfrac{d\sin\alpha \;\; \sin\beta}{Sin(\alpha+\beta)}$

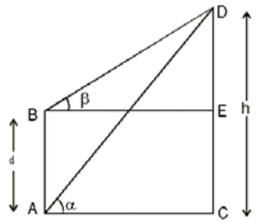
Where $\overline{AB} = \text{d}$

(or) $h = \dfrac{d}{Cot\alpha + \cot\beta}$



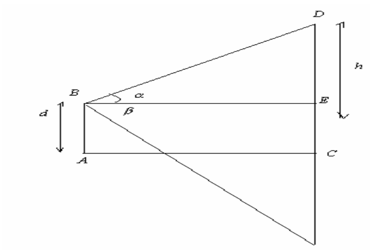
5. The angles of elevation of the top of a tower from the bottom and top of a building of height 'd' metres are β and α respectively. The height of the tower is

h = $\dfrac{d \sin\beta \cos\alpha}{\sin(\beta-\alpha)}$ metres (or) $h = \dfrac{d \cot\alpha}{\cot\alpha - \cot\beta}$



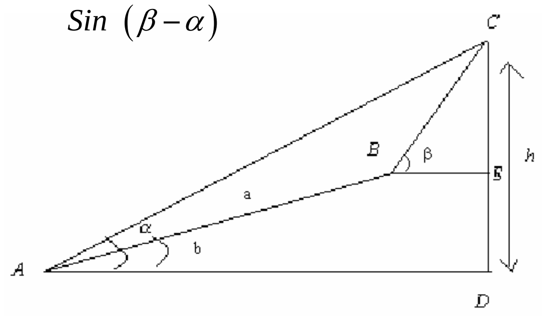
6. The angle of elevation of a cloud from a height 'd' metres above the level of water in a lake is 'α' and the angle of depression of its image in the lake is β. The height of the cloud from the water level in metres is

$h = \dfrac{d \sin(\beta+\alpha)}{\sin(\beta-\alpha)}$ **(or)** $h = \left[\dfrac{d\ (\tan\beta + \tan\alpha)}{(\tan\beta - \tan\alpha)}\right]$ **(or)** $h = d\left[\dfrac{Cot\ \alpha + Cot\beta}{Cot\alpha - Cot\ \beta}\right]$



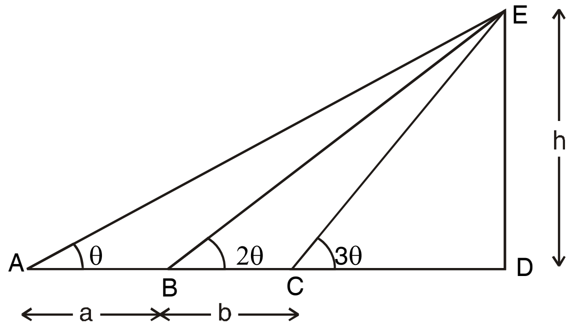
7. The angle of elevation of a hill from a point A is 'α'. After walking to some point B at a distance 'a' metres from A on a slope inclined at 'γ' to the horizon, the angle of elevation was found to be β.

Height of the hill $h = \dfrac{a\ Sin\ \alpha\ Sin\ (\beta-\gamma)}{Sin\ (\beta-\alpha)}$

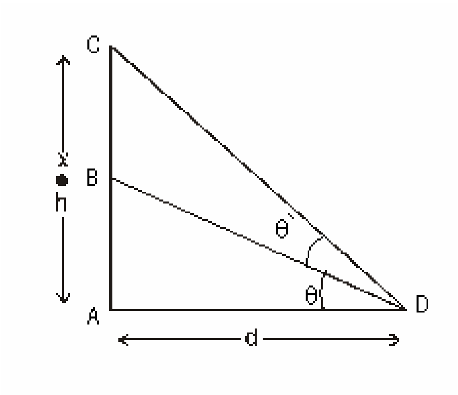


8. A balloon is observed simultaneously from the three points A, B, C on a straight road directly beneath it. The angular elevation at B is twice that at A and the angular elevation at 'C' is thrice that at A. If AB=a and BC=b then the height of the balloon h in terms of a and b is,

$$h = \frac{a}{2b}.\sqrt{(3b-a)\ (a+b)}$$



9. A flag staff stands on the top of a tower of height h metres. If the tower and flag staff subtend equal angles at a distance 'd' metres from the foot of the tower, then the height the flag - staff in metres is $h\left[\frac{d^2+h^2}{d^2-h^2}\right]$



HYPERBOLIC FUNCTIONS

Synopsis :

1. i) sinhx = $\frac{e^x - e^{-x}}{2}$

   ii) coshx = $\frac{e^x + e^{-x}}{2}$

   iii)tanhx = $\frac{e^x - e^{-x}}{e^x + e^{-x}}$

   iv)cothx = $\frac{e^x + e^{-x}}{e^x - e^{-x}}$

   v) sechx = $\frac{2}{e^x + e^{-x}}$

   vi)cosechx = $\frac{2}{e^x - e^{-x}}$ are called hyperbolic functions.

   Note : sin(ix) = isinhx, cos(ix) = coshx, tan(ix) = itanhx.

2. $\cosh^2 x - \sinh^2 x = 1$
3. $1 - \tanh^2 x = \text{sech}^2 x$
4. $\coth^2 x - 1 = \text{cosech}^2 x$
5. i) $\sinh(\alpha + \beta) = \sinh\alpha \cosh\beta + \cosh\alpha \sinh\beta$

   ii) $\sinh(\alpha - \beta) = \sinh\alpha \cosh\beta - \cosh\alpha \sinh\beta$

   iii)$\cosh(\alpha + \beta) = \cosh\alpha \cosh\beta + \sinh\alpha \sinh\beta$

   iv)$\cosh(\alpha - \beta) = \cosh\alpha \cosh\beta - \sinh\alpha \sinh\beta$

   v) $\tanh(\alpha + \beta) = \frac{\tanh\alpha + \tanh\beta}{1 + \tanh\alpha \tanh\beta}$

   vi)$\tanh(\alpha - \beta) = \frac{\tanh\alpha - \tanh\beta}{1 - \tanh\alpha \tanh\beta}$

6. i) sinh 2x = 2sinhx coshx

   ii) $\cosh 2x = \cosh^2 x + \sinh^2 x$

   iii)$\cosh 2x = 2\cosh^2 x - 1$ or $\cosh^2 x = \frac{1 + \cosh 2x}{2}$

   iv)$\cosh 2x = 1 + \sinh^2 x$ or $2\sinh^2 x = \frac{\cosh 2x - 1}{2}$

   v) $\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x}$

vi) $\sinh 3x = 3\sinh x + 4\sinh^3 x$

vii) $\cosh 3x = 4\cosh^3 x - 3\cosh x$

viii) $\tanh 3x = \dfrac{3\tanh x + \tanh^3 x}{1 + 3\tanh^2 x}$

7. **Values of inverse hyperbolic functions as logarithms functions** :

i) $\mathrm{Sinh}^{-1}x = \log_e(x + \sqrt{x^2 + 1})$

ii) $\mathrm{Cosh}^{-1}x = \log(x + \sqrt{x^2 - 1}),\ x \geq 1$

iii) $\mathrm{Tanh}^{-1}x = \dfrac{1}{2}\log\left(\dfrac{1+x}{1-x}\right),\ x \in (-1, 1)$

iv) $\mathrm{Coth}^{-1}x = \dfrac{1}{2}\log\left(\dfrac{x+1}{x-1}\right),\ |x| > 1$

v) $\mathrm{Sech}^{-1}x = \log\left(\dfrac{1 \pm \sqrt{1-x^2}}{x}\right),\ 0 < x \leq 1$

vi) $\mathrm{Co\,sec\,h}^{-1}x = \log\left(\dfrac{1+\sqrt{1+x^2}}{x}\right),\ x > 0$ or

$\log\left(\dfrac{1-\sqrt{1+x^2}}{x}\right),\ x < 0$